

## degli Studi di Padova

# INdAM Program on Serre Conjectures and the $p\mbox{-}adic$ local Langlands program

## Wednesday, May 15th

Pascal Boyer (LAGA) – 11.30 am, Aula 1BC/45. About Iharas lemma in higher dimension

In their work on Sato-Tate, Clozel-Harris-Taylor proposed a generalization of the Iharas lemma in higher dimension for some similitude groups. In this talk I will explain how to prove some particular cases through the study of the torsion in the cohomology of KHT Shimura varieties. In application we will also prove a level rising statement.

Thursday, May 16th

Peter Schneider (Münster)– 11.30 am, Aula 1BC/45.

## Support theory for the Iwahori-Hecke Ext-algebra

One technique to investigate abelian or triangulated categories is to establish a kind of geometric notion of support for their objects. This was originally developed by Gabriel for the abelian module category of certain rings. Eventually we want such a theory for the derived category of smooth representations of a p-adic Lie group in characteristic p. In this talk I will describe a first step in this direction, which is a support theory for the abelian category of graded modules over the corresponding cohomological Ext-algebra. The point is that this Ext-algebra does not satisfy Gabriel's assumptions.

Friday, May 17th

Peter Schneider (Münster)– 11.30 am, Aula 2BC/60. Support theory for the Iwahori-Hecke Ext-algebra -II

Monday, May 20th

Jacques Tilouine (LAGA)-10.00 am, Aula 2BC/30.

## Periods, Congruences and adjoint Selmer group for Bianchi modular forms

In a joint work with Eric Urban, we modified the periods of the differential forms associated to a Bianchi cusp form f which occurred in a formula for the congruence numbers of f, in order to obtain a Bloch-Kato-type formula for the rank three adjoint "motive of f". The modification takes into account the torsion of the Betti cohomology of a Bianchi threefold. This led us more recently to adapt a construction due to Galatius-Venkatesh in order to relate more directly the cohomology of this Bianchi threefold to the adjoint Selmer group of Bianchi modular forms.

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Pascal Boyer (LAGA)–11.30 pm, Aula 2BC/30. About Iharas lemma in higher dimension -II Tuesday, May 21st

Jacques Tilouine (LAGA) – 10.00 am, Aula 2BC/30. Periods, Congruences and adjoint Selmer group for Bianchi modular forms -II

Gabriel Dospinescu (ENS Lyon)-11.30 pm, Aula 2BC/30.

Integral p-adic cohomology of Drinfeld half-spaces -I

We will give an overview of the realization of the p-adic local Langlands correspondence for  $GL_2(Q_p)$  in the étale cohomology of the Drinfeld tower. This is joint work with Pierre Colmez and Wieslawa Niziol.

Wednesday, May 22nd

Wieslawa Niziol (CNRS-ENS Lyon)-15:00 am, Aula 1BC/45. Integral p-adic cohomology of Drinfeld half-spaces -II

I will compute the integral p-adic etale cohomology of Drinfeld half-spaces of any dimension. This refines the existing computation of the rational p-adic etale cohomology. The main tools are: the computation of the integral de Rham cohomology and the integral p-adic comparison theorems of Bhatt-Morrow-Scholze and Cesnavicius-Koshikawa which replace the quasi-integral comparison theorem of Tsuji used to compute the rational etale cohomology. This is a joint work with Colmez and Dospinescu.

Thursday, May 23nd

Pierre Colmez (CNRS-Jussieu)–11.30 am, Aula 1BC/45. Integral p-adic cohomology of Drinfeld half-spaces -III

Monday, May 27th

Benjamin Schraen (Orsay)-9.30-11.00 am, Aula 1 AD 100. p-adic automorphic forms -I

There exist interesting congruences between coefficients of modular forms. An explanation to this phenomenon is the existence of *p*-adic families of modular forms. The goal for this cours is to introduce the notion of *p*-adic modular forms and their arithmetic applications, especially in the etale cohomology of locally symmetric spaces.

Tony Feng (Stanford)-11.30 am, Aula 1 AD 100. The Galois action on p-adic symplectic K-theory Using the relation between integral symplectic groups and moduli of abelian varieties, one can equip p-adic symplectic K-theory with a Galois action. In joint work with Soren Galatius and Akshay Venkatesh, I compute this action and find that it enjoys an interesting universal property.

Sug Woo Shin (UC Berkeley)-14.30 pm, Aula 1 AD 100.

The local Langlands correspondence and local-global compatibility for GL(2) -I We will review the local Langlands correspondence for GL(n) over *p*-adic fields, with the n = 2 case as a key example, and describe the statement and proof of the local-global compatibility for GL(2)over **Q**.

Tuesday, May 28th

Benjamin Schraen (Orsay)–9.30-11.00 am, Aula 1 AD 100. p-adic automorphic forms -II

Karol Koziol (Alberta)-11.30 am, Aula 1 AD 100.

Serre weight conjectures for unitary groups

In the 1970s, Serre formulated his remarkable conjecture that every two-dimensional mod-p Galois representation of the absolute Galois group of  $\mathbf{Q}$ , which is odd and irreducible, should come from a modular form. He later refined his conjecture, giving a precise recipe for the weight and level of the modular form. Both the "weak form" and "strong form" of Serres conjecture are now theorems, due to the work of many mathematicians (Khare-Wintenberger, Kisin, Edixhoven, Ribet, and others). In this talk, we will discuss how to generalize Serres weight recipe when the Galois representation is replaced by a homomorphism from an absolute Galois group to the Langlands dual of a rank 2 unitary group. This is joint work with Stefano Morra.

Sug Woo Shin (UC Berkeley)-2.30-4.00 pm, Aula 1 AD 100. The local Langlands correspondence and local-global compatibility for GL(2) -II

Wednesday, May 29th

Benjamin Schraen (Orsay)–9.30-11.00 am, Aula 1 AD 100. p-adic automorphic forms -III

Jessica Fintzen (Cambridge-Michigan)-11.30 am, Aula 1 AD 100.

**Representations of p-adic groups** 

In the 1990s Moy and Prasad revolutionized the (complex and mod  $\ell$ ) representation theory of p-adic groups by showing how to use Bruhat-Tits theory to assign invariants to representations of p-adic groups. The tools they introduced resulted in rapid advancements in both representation theory and harmonic analysis – areas of central importance in the classical Langlands program. A

crucial ingredient for many results is an explicit construction of (types for) representations of p-adic groups. In this talk I will indicate why, survey what constructions are known and present recent developments based on a refinement of Moy and Prasad's invariants.

#### Florian Herzig (University of Toronto)-2.30 pm, Aula 1 AD 100.

*p*-modular and locally analytic representation theory of *p*-adic groups -I

This course will give an introduction to the mod p and p-adic representation theory of a p-adic reductive group (like  $\operatorname{GL}_n(\mathbf{Q}_p)$ ). Such representations naturally appear in the p-adic Langlands program. In the first part of the course we focus on smooth representations over  $\mathbf{F}_p$ , as first studied by Barthel-Livne. In the second part we discuss locally analytic representations over a p-adic field, as initiated by Schneider-Teitelbaum.

#### Thursday, May 30th

Sug Woo Shin (UC Berkeley)-9.30-11.00 am, Aula 1 AD 100. The local Langlands correspondence and local-global compatibility for GL(2) -III

Chan-Ho Kim (KIAS)-11.30 am, Aula 1 AD 100.

On the quantitative variation of congruence ideals of modular forms

We discuss a quantitative version of Ribet's level lowering theorem for modular forms of higher weight. This is joint work with Kazuto Ota.

Florian Herzig (University of Toronto)-2.30-4.00 pm, Aula 1 AD 100. p-modular and locally analytic representation theory of p-adic groups -II

Friday, May 31st

Sug Woo Shin (UC Berkeley)-9.30-11.00 am, Aula 1 AD 100. The local Langlands correspondence and local-global compatibility for GL(2) -IV

#### Michael Schein (Bar-Ilan)-11.30 am, Aula 1 AD 100.

Supersingular mod *p* representations of *p*-adic groups

The lecture will survey what is known about the supersingular representations of p-adic reductive groups. These are the building blocks of the mod p smooth representation theory of such groups, as discussed in Florian Herzig's course. We will discuss work of numerous mathematicians giving explicit classifications, where known, as well as some partial constructions and more general results. We will also consider the analogous theory in the category of Iwahori-Hecke modules.

Florian Herzig (University of Toronto)-2.30pm-4.00pm, Aula 1 AD 100. p-modular and locally analytic representation theory of p-adic groups -III

#### Monday, June 3rd

## Sug Woo Shin (UC Berkeley)-9.30-11.00 am, Aula 1 AD 100. The local Langlands correspondence and local-global compatibility for GL(2) -V

#### Koji Shimizu (Berkeley)-11.30 am, Aula 1 AD 100.

#### Constancy of generalized Hodge-Tate weights of a *p*-adic local system

A *p*-adic local system on a rigid analytic variety can be regarded as a family of local Galois representations parametrized by the variety. In this talk, we will discuss the constancy of generalized Hodge-Tate weights of a *p*-adic local system. This is one instance of rigidity phenomena of geometric families of Galois representations in contrast to arithmetic families in the Galois deformation theory.

Florian Herzig (University of Toronto)-2.30-4.00 pm, Aula 1 AD 100. p-modular and locally analytic representation theory of p-adic groups -IV

#### Tuesday, June 4th

#### Yongquan Hu (Morningside)-10.00 am, Aula 1 AD 100.

On the mod p cohomology of Shimura curves

The mod p local Langlands correspondence is well-understood for  $GL_2(Q_p)$ , but is still very mysterious in other cases. In this talk, I will discuss some results on the mod p correspondence for  $GL_2(F)$  when F is a finite unramified extension of  $Q_p$ , in the context of the Buzzard-Diamond-Jarvis conjecture. This is joint work in progress with Haoran Wang.

#### David Savitt (Johns Hopkins)-11.30 am, Aula 1 AD 100.

#### Moduli of potentially Barsotti–Tate Galois representations

In this talk I will explain how the geometry of Galois moduli stacks is related to the weight part of Serre's conjecture for GL(2) and to the geometric Breuil-Mezard conjecture for potentially Barsotti-Tate representations. This is joint work with Ana Caraiani, Matthew Emerton, and Toby Gee.

## Ana Caraiani (Imperial College London)-2.30 pm, Aula 1 AD 100.

#### On the geometry of the Hodge-Tate period morphism

In this talk, I will describe joint work with Peter Scholze on the geometry of the Hodge–Tate period morphism for perfectoid Shimura varieties. I will focus on how to relate the fibers of this morphism to perfectoid Igusa varieties, for the open Shimura variety but also for its minimal and toroidal compactifications. I will also mention applications to the cohomology of Shimura varieties and beyond.

#### Matthew Emerton (University of Chicago)-4.00 pm, Aula 1 AD 100.

Localizing  $GL_2(\mathbf{Q}_p)$ -representations

We let Z denote a certain chain of projective lines over a field k of characteristic p, and we let A denote the category of smooth (but not necessarily admissible)  $\operatorname{GL}_2(\mathbf{Q}_p)$ -representations on k-vector spaces (with some fixed central character). Our goal in this talk is to explain how the category A may be "localized" over the scheme Z, and to explain various applications of this localization.

This is work-in-progress with Andrea Dotto and Toby Gee.

## Wednesday, June 5th

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## Eugen Hellmann (University of Muenster)-9.30-11.00 am, Aula 1 AD 100. p-adic Hodge theory and deformations of Galois representations -I

John Enns (UBC)-11.30 am, Aula 1 AD 100.

Aspects of mod p local-global compatibility

Let F be a CM number field. Given a global automorphic mod p Galois representation  $\rho: G_F \to \operatorname{GL}_n(\overline{\mathbb{F}}_p)$  and a choice of place w dividing p, there is a naturally associated smooth  $\operatorname{GL}_n(F_w)$ -representation  $\Pi(\rho)$  over  $\overline{\mathbb{F}}_p$  constructed using mod p modular forms with infinite level at w. It is an open problem to determine whether  $\Pi(\rho)$  depends only on the local factor of  $\rho$  at w (i.e. mod p local-global compatibility). In this talk I will discuss the "other direction" to this problem, namely how the data of  $\rho|_{G_{F_w}}$  gets encoded into  $\Pi(\rho)$  in cases of interest. This involves the equality of some local mod p Galois parameters defined using potentially crystalline lifts of  $\rho|_{G_{F_w}}$  and local automorphic parameters defined using certain mod p Hecke operators.

Florian Herzig (University of Toronto)-2.30-4.00 pm, Aula 1 AD 100. p-modular and locally analytic representation theory of p-adic groups -V

## Thursday, June 6th

Stefano Morra (Université Paris 8–LAGA)–9.30-11.00 am, Aula 1 AD 100. Patching and the local Langlands correspondence -I

Federico Bambozzi (Oxford)-11.30 am, Aula 1 AD 100. A global perspective on Hodge Theory

In this talk I give a brief and informal explanation of how to define analytic spaces over any Banach ring. I then present some examples related to arithmetic, in particular to p-adic Hodge Theory. This is a work in progress with Kobi Kremnizer and Adam Topaz.

Eugen Hellmann (University of Muenster)-2.30-4.00 pm, Aula 1 AD 100. p-adic Hodge theory and deformations of Galois representations -II

Friday, June 7th

Stefano Morra (Université Paris 8-LAGA)-9.30-11.00 am, Aula 1 AD 100.

#### Patching and the local Langlands correspondence -II

Vlad Serban (Vienna)–11.30 am, Aula 1 AD 100.

 $\ensuremath{\textit{p}}\xspace$  and applications and applications

It was conjectured by Manin and Mumford and established by work of Raynaud and many others that subvarieties of a semi-abelian variety G which intersect more than finitely many torsion points of G must contain a translate of a semi-abelian subvariety of G. We present a p-adic version of this result for formal subschemes of a p-divisible formal Lie group G. We then outline some applications, including to non-vanishing for certain two-variable p-adic L-functions.

Benjamin Schraen (Orsay)–2.30-4.00 pm, Aula 1 AD 100. p-adic automorphic forms -IV

Monday, June 10th

Stefano Morra (Université Paris 8–LAGA)–9.30-11.00 am, Aula 1 AD 100. Patching and the local Langlands correspondence -III

Konstantin Ardakov (Oxford)-11.30 am, Aula 1 AD 100.

#### The first Drinfeld covering and equivariant *D*-modules on rigid spaces

Let p be a prime and let F be a p-adic local field. The p-adic upper half plane Omega is obtained from the projective line viewed as a rigid analytic variety by removing the F-rational points. Drinfeld introduced a tower of finite etale Galois coverings of Omega by interpreting Omega as the rigid generic fibre of the moduli space of certain formal one-dimensional commutative groups with quaternionic multiplication, and introducing level structures to define the coverings. This tower is now known to realise both the Jacquet-Langlands and local Langlands correspondences for  $G = GL_2(F)$  in  $\ell$ -adic etale cohomology, where  $\ell$  is a prime not equal to p. Coherent cohomology of the tower is expected to produce representations of G which are admissible in the sense of Schneider and Teitelbaum. Using the theory of equivariant D-modules on rigid spaces we can prove that the dual of the global sections of a non-trivial line bundle arising from the first covering of Omega is an irreducible admissible representation of G. Patel, Schmidt and Strauch have also given an argument for the admissibility of these representations using a formal model for the first covering; whilst similar in certain respects, our approach is significantly different to theirs. This is joint work with Simon Wadsley.

Benjamin Schraen (Orsay)-2.30-3.40 pm, Aula 1 AD 100. p-adic automorphic forms -V

Tuesday, June 11th

#### Fabrizio Andreatta (Milano)-10.00 am, Aula 1 AD 100.

#### Katz type p-adic L-functions for primes p non-split in the CM field

I will discuss a way to construct anticyclotmic p-adic L-functions attached to an elliptic eigenform f and an imaginary quadratic field K, interpolating p-adically the central critical values of the Rankin L-functions of f twisted by anticyclotomic characters of higher infinity type. I will also provide p-adic Gross-Zagier formulae for these p-adic L-functions. Here the prime p is assumed to be inert or ramified in K. The case that p is split is due to Katz (for Eisenstein series) and Bertolini-Darmon-Prasana (for cuspidal eigenforms). This is joint work with Adrian Iovita.

#### Sandra Rozsenstajn (ENS-Lyon)-11.30 am, Aula 1 AD 100.

#### On the reduction modulo p of crystalline representations of dimension 2

I will talk about the problem of studying the reduction modulo p of crystalline representations of dimension 2 of the Galois group of  $\mathbf{Q}_p$ . In particular, I will be interested in the following situation: fix Hodge–Tate weights and a residual representation, and consider the locus parametrizing crystalline representations with the given weights and reduction modulo p. What can be said about this locus in general?

## Gergely Zabradi (Budapest)–2.30 pm, Aula 1 AD 100. Multivariable ( $\varphi, \Gamma$ )-modules

The notion of cyclotomic multivariable  $(\varphi, \Gamma)$ -modules were introduced recently in order to generalize (parts of) Colmez's work on the *p*-adic Langlands programme from  $GL_2(\mathbf{Q}_p)$  to groups of higher rank. More specifically: there exists a functor with promising exactness- and compatibility properties from the category of smooth mod  $p^n$  representations of the group G of  $\mathbf{Q}_p$ -points of a  $\mathbf{Q}_p$ -split reductive group with connected centre to d-variable  $(\varphi, \Gamma)$ -modules where d is the number of simple roots of G. Further, there is a Fontaine-style equivalence of categories between these multivariable objects and *p*-adic representations of d-fold products of local Galois groups. There is a new proof of this fact using Drinfeld's lemma for perfectoid spaces (jt. with Annie Carter and Kiran S. Kedlaya). Usual methods like overconvergence and Herr's complex computing cohomology generalize to this context, too. In part also joint work with Aprameyo Pal.

#### Laurent Berger (ENS-Lyon)-4.00 pm, Aula 1 AD 100.

#### Tensor products and trianguline representations

Two *p*-adic Galois representations whose tensor product is trianguline are themselves potentially trianguline. I will recall the definition of trianguline representations and explain the proof of the above statement.

#### Wednesday, June 12th

Stefano Morra (Université Paris 8–LAGA)–9.30-11.00 am, Aula 1 AD 100. Patching and the local Langlands correspondence -IV

Tobias Schmidt (Rennes)-11.30 am, Aula 1 AD 100. Mod p Hecke algebras and dual equivariant cohomology Let H be Vignéras pro-p Iwahori-Hecke algebra with mod p coefficients of a p-adic GL<sub>n</sub>. Its finite part acts on the cohomology of the mod p dual flag variety by means of classical Demazure operators. We explain how this finite representation can be uniquely extended to a natural representation of the whole algebra H and give some of its properties. Compatibility of the construction with various operations on the dual group give applications to the mod p local Langlands correspondence. If time permits, we discuss the cases of n = 2 and n = 3 in some detail. This is joint work with C. Ppin.

Eugen Hellmann (University of Muenster)-2.30-4.00 pm, Aula 1 AD 100. p-adic Hodge theory and deformations of Galois representations -III

#### Thursday, June 13th

Stefano Morra (Université Paris 8–LAGA)–9.30-11.00 am, Aula 1 AD 100. Patching and the local Langlands correspondence -V

Daniel Le (Toronto)-11.30 am, Aula 1 AD 100.

The mod p cohomology of Shimura curves at first principal congruence level

It is expected that the mod p completed cohomology groups of Shimura curves realize a hypothetical mod p Langlands correspondence. Breuil and Paskunas constructed combinatorial representationtheoretic objects called Diamond diagrams (close relatives of Hecke modules) and conjectured that some of them appear in the cohomology of Shimura curves at first principal congruence level. This conjecture was proven in groundbreaking work of Emerton–Gee–Savitt. We will present a refinement of this result. Namely, we determine the diagram structure of the generic part of this cohomology in terms of the local Galois action. We also prove a conjecture of Breuil relating Diamond diagrams and Galois representations using a combinatorial generalization of Colmez's functor. We will emphasize the role played by the Taylor–Wiles method in the above results. Some of the results discussed are part of joint works with A. Dotto, S. Morra, and B. Schraen and some were obtained independently by Y. Hu and H. Wang.

Eugen Hellmann (University of Muenster)-2.30-4.00 pm, Aula 1 AD 100. p-adic Hodge theory and deformations of Galois representations -IV

## Friday, June 14th

Eugen Hellmann (University of Muenster)-9.30-11.00 am, Aula 1 AD 100. p-adic Hodge theory and deformations of Galois representations -V

Zijian Yao (Harvard)–11.30 am, Aula 1 AD 100. A crystalline perspective on Ainf-cohomology Ainf-cohomology theory is a cohomology theory constructed by Bhatt–Morrow–Scholze that interpolates various p-adic cohomology theories (notably etale, crystalline and de Rham cohomology). In this talk we review the (classical) construction of this theory, and provide a crystalline interpretation of Ainf-cohomology valued on certain "small" object. This allows us to connect Ainf-cohomology to the absolute crystalline cohomology valued in Fontaine's period ring Acris, and recover various p-adic comparison theorems. If time permits, we briefly mention the extension to the logarithmic

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setup.